1899MNRAS..59..513P

Longitude from Moon Culminations. By D. A. Pio.

(*Communicated by the Secretaries.*)

A. *Theory.*

Purpose of the New Method.—For the determination of longitude on land, especially during journeys through the continents of Africa and Australia, as also on touching an unknown island in the southern seas, the author dares to express his hope that the method put forward in this paper is preferable to the obsolete one of lunar distances.

In the new method the culmination of the Moon serves only to determine exactly, easily, and quickly the precise instant of the Moon's passage through the local meridian. The longitude is deduced from this instant as in the method of Moon's transits. The practical advantage of the new method is that it does not require any transit instruments, and dispenses with the laborious setting of a telescope exactly in the meridian.

The observer is supposed to possess a good sextant furnished with a telescope magnifying at least twice, an artificial horizon, a *well-rated* chronometer, the *Nautical Almanac* for the current year, a set of nautical and logarithmic tables, and last, but not least, to operate on good solid ground, with an assistant to note down the instants of the different observations.

Remarks on Culminations.—The author would draw the attention of the reader to the sense of the word "culmination" as used in this paper. The author does *not* mean by it that the centre of the celestial body is exactly on the local meridian, but that this body is at its greatest altitude. Whether the greatest altitude coincides or not with the meridian passage is quite another question. The author reminds the reader that only the fixed stars culminate really in the meridian. The Sun, the Moon, and all the planets culminate *out* of the meridian.

"Meridian passage" or "transit" is the word the author uses in this paper instead of "culmination," when he means that the centre of the celestial body is exactly on the meridian.

Principle of the New Method.—The local longitude is found very simply through the right ascension of the Moon at the instant of her transit at the place of observation. This right ascension furnishes the corresponding mean time of Greenwich, and the difference between this time and the mean local time at Moon's transit is the longitude sought, in time. So far, there is nothing new.

In the new method the right ascension is not found directly by observation, but deduced from the mean local time of Moon's meridian passage. For this time converted into sidereal time and added to the right ascension of the mean Sun at his transit

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in the place of observation, gives immediately the sought right ascension of the Moon.

The real difficulty, therefore, is to find exactly the mean local time at Moon's transit without using a transit instrument. The culmination of the Moon out of the meridian is substituted for her meridian passage, and the lapse of time between transit and culmination is found by a calculation which the author calls "reduction to the meridian." Thus, the instant of Moon's culmination gives the instant of Moon's transit.

The culmination of the Moon can be observed with a good sextant, and so the heavy transit instrument and the difficult operations required to place the instrument exactly in the meridian are dispensed with. However, the culmination can *not* be observed directly, and the author deduces the exact instant of meridian passage from the instants at which two equal altitudes of the Moon are taken. The middle time between these two instants is used in the calculation of reduction to the meridian, instead of the precise instant of culmination.

The instant of Moon's meridian passage becomes therefore known when the instants of two equal altitudes are given in mean local time. This local time requires the determination of an hour angle from an altitude of the Sun, for the observer possesses only a chronometer indicating, more or less exactly, mean Greenwich time.

This hour angle can be dispensed with, and all errors consequent upon it can be totally eliminated by simply determining the instant of Sun's culmination in the place of observation. This is done, as above in the case of the Moon, by taking two equal altitudes of the Sun. The middle gives then, by reduction to the meridian, the time of Sun's meridian passage, which must be corrected by the equation of time in order to give the instant of mean noon at the place of observation.

When the instants of Moon's transit and of mean Sun's transit have been deduced by calculation from the instants, as given by the chronometer, of the equal altitudes both of Sun and Moon, the mean local time of Moon's transit is given by subtraction.

The novelty in the new method consists, therefore, in the use of the method of equal altitudes. This use renders unnecessary the determination of the meridian's position, the employment of complicated instruments, the observation of Moon culminating stars, and the calculation of hour angles. The arcs measured with the sextant are not used at all in the calculation. All is reduced to the indications of the chronometer.

The practical difficulty in this method consists only in the perfect determination of the instants at which the different equal altitudes are taken.

How to Make the Observations Precise.—In order to obtain the longitude with accuracy the lapse of time between Sun's

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transit and Moon's transit must be correct down to the tenth of a second, as even this small fraction corresponds to a mean error in longitude of two-thirds of a minute (of arc). The first requirement is, therefore, that the rate of the chronometer be as uniform as possible, and that its daily amount be ascertained with the greatest precision. In consequence the two culminations of the Sun and the Moon ought to be as near to each other as possible, and to be chosen so that the interval between them be *never* more than twelve hours.

The instants of the different observations must be given to tenths of seconds, which can be done very easily by the chronograph. However, the difficulty is not in noting down the instant of observation, but in catching the precise instant at which the Sun or Moon is *exactly* at the altitude indicated by the graduation. In order to facilitate this the observation must be made with the artificial horizon, and the telescope of the sextant must have a certain magnifying power—say, about five. In this way the contact of the direct image with the reflected one can be observed with more distinctness, and the real instant of contact is perceived with fewer chances of error.

Every error in the instant of observation will influence the middle time deduced therefrom, and in order to give to these times a greater degree of exactness four pairs of equal altitudes must be taken for the Sun and as many for the Moon. The mean of the four middle times for the Sun (or the Moon) may be trustworthy.

Besides, the altitudes of both Sun and Moon must be such that a perceptible change in altitude takes place in so small an interval of time as a tenth of a second. Therefore, the altitudes must be low, but not so much that variations in refraction may make unequal altitudes appear equal.

For this reason the new method cannot be used with great success in high latitudes.

Reduction to the Meridian.—The lapse of time between meridian passage and middle time is given by a well-known formula, which all treatises on nautical astronomy bring under the head 'time from equal altitudes.' The author uses this formula under the following form :

$$x'' = +\frac{\delta_1'' - \delta_2''}{2 \sin H}\left\{ \cot \frac{\delta_1 + \delta_2}{2} . \cos H - \tan \lambda \right\},$$

where x'' is the angle in seconds at the pole between the meridian and that circle of declination which bisects the angle 2H ;

2H the angle at the pole between the circle of declination of the Sun or the Moon at the first observation, and that of the same celestial body at the second observation ;

δ_1 the polar distance from the elevated pole of the observed celestial body at first observation ;

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δ_2 the polar distance of the same body at second observation ;
λ the latitude of the place of observation ;
$\delta_1'' - \delta_2''$ is the difference in seconds between the polar distances.

The angle x'' must now be divided either by 15″ in the case of the Sun, or by $15''{\cdot}0411 - \frac{\Delta^s}{40}$ in the case of the Moon, in order to give the lapse of time between meridian passage and middle time. The symbol Δ^s stands for the variation in 10^m of the Moon's right ascension, as given by the *Nautical Almanac* for the Greenwich time at Moon's transit.

The difference $\frac{\delta_1'' - \delta_2''}{2}$ must be calculated with the greatest precision, as any error in it has an influence on the right ascension of the Moon from which the longitude is obtained.

The angle 2 H is deduced from the interval of time between the observations of the two equal altitudes. 2 H = this interval in seconds, multiplied by 15″ in the case of the Sun, or multiplied by $15''{\cdot}0411 - \frac{\Delta^s}{40}$ in the case of the Moon.

Error in Longitude.—As the variation in 10^m of Moon's right ascension oscillates from 17^s to 29^s, the error in longitude corresponding to one second of right ascension oscillates from $\frac{150'}{17} = 8'{\cdot}8$ to $\frac{150'}{29} = 5'{\cdot}2$, the mean being $7'$. The sources of error are :

1. Wrong middle time, arising from want of correspondence between the altitudes measured by the sextant and the indications of the chronometer.
2. Wrong reduction to the meridian arising from wrong values of latitude and polar distances being used in the calculation.
3. Wrong lapse of time between the transit of the Sun and that of the Moon, arising from wrong value of chronometer's daily rate.

The author makes thereon the following remarks :

To 1. The use of four pairs of equal altitudes combined with the employment of the artificial horizon and the magnifying power of five renders the mean of the middle times exact to one-tenth of a second at least.

To. 2. When the quantity $\frac{\delta_1'' - \delta_2''}{2}$ is calculated exactly, and that can always be done, the wrong values of latitude and of the polar distances have no influence on the reduction to the meridian.

To 3. Nowadays chronometers are so well constructed that the changes in the daily rate are insignificant ; besides, the influence of temperature is now taken into account.

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Therefore the author hopes that the error in longitude by the new method will amount only to a few minutes of arc in unfavourable circumstances.

Longitude by the Sun.—As in the new method the instant of the mean Sun's transit at the place of observation is given in mean Greenwich time, the local longitude in time is evidently the difference between 12^h and this Greenwich time.

Therefore the longitude is furnished by the new method in two different ways:

1. Longitude by the Sun, deduced directly from the chronometer;
2. Longitude by the Moon, deduced from her right ascension.

Comparison of the new method with the known ones.—The author has occupied himself during many years with finding out a new method for the determination of longitude and tried many contrivances of his own. He has come to the following conclusions:

1. On sea the *best* method is to determine longitude by the chronometer compared with local time deduced from altitudes of the Sun.
2. On sea the method of lunar distances is still the *best* when one wishes absolutely to use the Moon for determining the longitude on board a ship. All methods proposed to supersede Borda's one are inferior to it, and it is lost time and wasted ingenuity to devise such new methods.
3. On sea Borda's method cannot compete with longitude by chronometer, especially now that chronometers have so much improved.
4. On land the telegraph is unsurpassable in its accuracy for the purpose of determining longitudes.
5. On land the method of deducing the right ascension of the Moon from comparison with moon-culminating stars comes the next after the telegraphic method. Its inconveniences consist:
    1. In the necessity of determining accurately the local meridian.
    2. In the use of heavy instruments.
    3. The delicacy of the observations, and the many corrections required.
6. The new method requires no meridian, may be used very easily and everywhere on land, and rests on the indications of the chronometer only. The measurements with the sextant are only used to fix the instants of the observations. The new method is properly a chronometrical method.

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7. On land the longitude by the Sun, as determined by the new method from equal altitudes of the Sun, is, when a well-rated chronometer is used, the simplest of all methods after the telegraphic one.
8. The longitude by the Moon, according to the new method, is a good check on the chronometer.

B. *Practice.*

Directions for the observer.—(1). Do not use this method when your latitude exceeds sixty degrees north or south. (2) Determine your latitude with the greatest precision by the artificial horizon, if possible, by the means of different observations. (3) Choose such a day that the Moon's daily variation in right ascension be as great as possible. (4) With the best assumed longitude calculate the approximate Greenwich time of Moon's meridian transit. (5) Before culmination take four altitudes of the Moon in succession, from three to three minutes, and as many corresponding ones after culmination. (6) The observed altitudes of the Moon must not be less than thirty degrees, nor more than sixty. (7) The time of taking the altitudes of the Moon must not be less than one hour and a half before and after culmination, nor more than two hours before and after it. (8) For the *nearest* Sun's meridian transit take four successive altitudes of the Sun before noon, as near the prime vertical as possible, and as many corresponding ones after noon. (9) All altitudes must be taken with the artificial horizon. (10) The instant of every observation must be taken with a chronograph to tenths of seconds.

Rules for the calculations.—(1) First calculate longitude by the Sun, then longitude by the Moon. (Note—These two calculations are to be effected in the same manner, except where the rules expressly indicate the contrary.) (2) Calculate the mean time of the four observations before culmination and that of the corresponding ones after culmination. From these two means deduce the middle time. (3) Find the difference between these two means and convert it into seconds. In the case of the Sun multiply the half of the found number of seconds by 15 and call the result H. (3 *bis*) In the case of the Moon find first in the *Nautical Almanac* the variation in 10^m of the Moon's right ascension for the hour of middle time, divide this number by 40 and subtract the quotient from $15''.0411$. The result is the factor of the half difference between the means. (4) Calculate (down to tenths of seconds) the polar distance from the elevated pole of the Sun (or Moon) for the instant given by the mean of the observations *before* culmination and call it δ_1. (5) Calculate (down to tenths of seconds) the polar distance from the elevated pole of the Sun (or Moon) for the instant given by the mean of

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the observations *after* culmination and call it δ_2. (6) Take the half *sum* of these two polar distances and call the result $\frac{\delta_1+\delta_2}{2}$. (7) Take the half *difference* of these two polar distances, express the result in seconds, and call it $\frac{\delta_1''-\delta_2''}{2}$. (NOTE I.—In the case of the Moon it is necessary to verify the amount $\frac{\delta_1''-\delta_2''}{2}$ by calculating separately the variation of Moon's declination during the interval of time elapsing from the mean instant of the observations before culmination to the mean instant of the observations after culmination. NOTE II.—Particular attention must be paid to the *sign* of $\frac{\delta_1''-\delta_2''}{2}$.) (8) For longitude by the Sun calculate the equation of time for the instant of Sun's transit at the place of observation, with the help of the longitude by account. (9) Calculate the reduction to the meridian by the rules in Article 11 and pay particular attention to the *sign* of the result. (10) To the middle time, as found by Rule 2, apply the above reduction to the meridian with its *proper* sign. (NOTE—In the case of the Sun apply also the equation of time with its *proper* sign.) (11) For longitude by the Sun call the result 'Greenwich time of mean Sun's transit,' and find the difference between it and 12^h. This difference is the sought longitude. (12) For longitude by the Moon call the sum of middle time and reduction to the meridian 'Greenwich time of Moon's transit,' and find the difference between it and that of mean Sun's transit. The result is the mean local time of Moon's transit. Mark whether it is a.m. or p.m. (13) Convert the local time of Moon's transit into the equivalent sidereal time (down to hundredths of seconds) and call the result 'Moon's difference in right ascension.' (14) Calculate (down to hundredths of seconds) the right ascension of the mean Sun at the Greenwich time of mean Sun's transit in the place of observation. (15) To this right ascension of the mean Sun *add* the sidereal time calculated by Rule 13, if the local time of Moon's transit is p.m.; *subtract* this sidereal time from the above said right ascension of the mean Sun when the said local time is a.m. The result is the Moon's right ascension at the instant of Moon's transit. (16) Calculate (with the help of the *Nautical Almanac*) the Greenwich time corresponding to this right ascension of the Moon. The difference between this Greenwich time and the local time of Moon's transit is the sought longitude in time.

Rules for reduction to the meridian.—(1) Take the logarithm of cosine H, that of cotangent $\frac{\delta_1+\delta_2}{2}$, add them, and find the *number* corresponding to the sum. If $\frac{\delta_1+\delta_2}{2}$ is less than 90° this number is positive; if it is more than 90°, negative.

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(2) From this number subtract *algebraically* the *natural* tangent of latitude and give to the result its *proper* sign. (3) Take the logarithm of this result, that of $\frac{\delta_1'' - \delta_2''}{2}$ as expressed in seconds, the arithmetical complement of sine H and the arithmetical complement of 15″ for the Sun, or that of $15''\cdot0411 - \frac{\Delta^s}{40}$ for the Moon, add all together and find the *number* corresponding to the sum. This number is the reduction to the meridian in seconds of time. Pay particular attention to its time. (NOTE I.—It is good to write the words 'positive' or 'negative' opposite the different numbers entering into this calculation. NOTE II.—For the Sun *four* decimal places are sufficient for the abovesaid logarithms. For the Moon *five* decimals are necessary.)

Example.

1885 March 27, in a place on the island of Syra, situated exactly in latitude 37° 25′ 30″ N., but in longitude by account 25° E., the following observations were taken with the artificial horizon:

Measured Heights.		Instants of Observation.
	° ′ ″	h m s
1. Sun's L.L.	78 36 0 ...	7 34 50·9 a.m.
2. "	78 36 0 ...	0 55 10·7 p.m.
3. Moon's L.L.	107 44 20 ...	6 34 33·4 p.m.
4. "	107 44 20 ...	9 37 36·2 p.m.

The above times are in Greenwich time, as given by a chronometer whose indications have been corrected for error and daily rate.

		h m s		° ′ ″
Middle time	=	10 15 0·8 a.m.		
Half difference (= H)	=	2 40 9·9	=	40 2 28·5
Polar distance of Sun at first observation ($=\delta_1$)			=	87 17 39·2
Polar distance of sun at second observation ($=\delta_2$)			=	87 12 26·5
$\frac{\delta_1+\delta_2}{2}$			=	87 15 02·9
$\frac{\delta_1-\delta_2}{2}$			=	2 36·35

Reduction to the meridian calculated by above formula ... ... $= -11^s\cdot8$

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	h m s
Middle time	10 15 00·8 a.m.
Reduction to the meridian	−11·8
Equation of time	5 23·0
Greenwich time of mean Sun transit	10 20 12·0 a.m.
Local time	12 0 0·0
Difference	−1 39 48
	° ′ ″
Longitude of Syra by the Sun	24 57 0·0

Calculation of Longitude by the Moon.

I. *Preliminary Calculations.*

	h m s
Time of Moon's first observation	6 34 33·4 p.m.
Time of Moon's second observation	9 37 36·2 „

		h m s		° ′ ″
Middle time	=	8 06 04·8 p.m.		
Half difference	=	1 31 31·4	=	22 5 8
Polar distance of Moon at first observation $(=\delta_1)$			=	81 27 51·5
Polar distance at second observation $(=\delta_2)$			=	81 57 32·8
$\frac{\delta_1+\delta_2}{2}$			=	81 42 42·2
$\frac{\delta_1-\delta_2}{2}$			=	−14 50·65

Reduction to the meridian calculated by the above formula ... = $1^{m}\ 43^{s}\cdot14$

	h m s
Middle time	8 6 04·80 p.m.
Reduction to the meridian	1 43·14
Moon's transit in Syra (Greenwich time)	8 7 47·94 p.m.
Mean Sun's transit in Syra „	10 20 12·00 a.m.
Moon's transit in Syra (Syra time)	9 47 35·94 p.m.
Acceleration for $9^{h}\ 47^{m}\ 36^{s}$	1 36·53
Sidereal time at Sun's transit in Syra	20 3·05
Moon's right ascension at transit at Syra	10 9 15·52

II. *Calculation of Longitude.*

Moon's right ascension for March 27, 8^{h} is ...	10 8 57·88
Change in $7^{m}\ 50^{s}\cdot4$ is	17·64
Therefore at Greenwich time $8^{h}\ 07^{m}\ 50^{s}\cdot4$ the Moon's R.A. is	10 9 15·52

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	h m s
Moon's transit (Syra time)	9 47 35·9 p.m.
„ (Greenwich time)	8 7 50·4 p.m.
Difference	1 39 45·5
Longitude of Syra by Moon...	24° 56′ 22″·5 E.

Syra, Greece:
1898 *November* 2.

Errata.

Professor Schur's paper on *The Diameter and Compression of the Planet Mars.*

Page 330, first line of table, mean of h and v, *for* 6″·23 *read* 6″·28.
„ 331, third line from bottom, should *read* $2a$, 6·370; $2b$, 6·275; Diff. 0·095; $a = \frac{a-b}{a}$, 67

Cape Observations of Nebulæ: Page 339, the paragraph commencing h 262 should run on with next paragraph, and *read*—

. . . and is 13′ N. p h 2630 = G.C. 838 . . .

Cape Observations of Occultations, page 340, lines 11, 12—

7-in. equatorial, δ lat., *for* − *read* +.
10-in. astrographic telescope, δ lat., *for* + *read* −.

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